

# Quark-Hadron Duality in Neutron Spin-Structure and $g_2$ moments at intermediate $Q^2$

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**Abstract.** Jefferson Lab experiment E01-012 measured the  $^3\text{He}$  spin-structure functions and virtual photon asymmetries in the resonance region in the momentum transfer range  $1.0 < Q^2 < 4.0$  (GeV/c)<sup>2</sup>. Our data, when compared with existing deep inelastic scattering data, were used to test quark-hadron duality in  $g_1$  and  $A_1$  for  $^3\text{He}$  and the neutron. In addition, preliminary results on the  $^3\text{He}$  spin-structure function  $g_2$ , on the Burkhardt-Cottingham sum rule and on higher twist effects through the  $x^2$ -weighted moment  $d_2$  of the neutron were presented.

**Keywords:** Spin-Structure Functions, Quark-hadron Duality, Higher Twists

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## QUARK-HADRON DUALITY

In 1970, Bloom and Gilman [1] observed that structure function data taken at the Stanford Linear Accelerator Center (SLAC) in the resonance region average to the scaling curve of deep inelastic scattering (DIS). From the time the observation of quark-hadron duality was made, substantial efforts were put into a theoretical explanation for this phenomenon. In addition, the idea of a dual behavior between quarks and hadrons was extended to spin structure function  $g_1$ . Recent data from Jefferson Lab (JLab) [2, 3, 4, 5] and DESY [6] on the proton in the resonance region indicate the onset of duality at momentum transfers ( $Q^2$ ) as low as 0.5 and 1.6 (GeV/c)<sup>2</sup> for the unpolarized and polarized structure functions, respectively.

Carlson and Mukhopadhyay [7] showed within perturbative QCD that, at large  $Q^2$  and as  $x$  goes to 1, structure functions in the resonance region behaves the same way as in DIS region<sup>1</sup>. In the high  $x$  region, the photon is more likely to interact with the quark having the same helicity as the nucleon. This implies that both  $g_1$  and the unpolarized structure function  $F_1$  behave as  $(1-x)^3$  when  $x \rightarrow 1$ . The virtual photon-nucleon asymmetry  $A_1$  is expected [8] to tend to 1 as  $x \rightarrow 1$  in the scaling region. Carlson and Mukhopadhyay, considering resonant contributions and non-resonant background, predict the same behavior in the resonance region at large enough momentum transfer. Recently, Close and Melnitchouk [9] studied three different conditions of SU(6) symmetry breaking in the

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<sup>1</sup> In this proceeding, we call  $x$  the Björken variable which is defined, in the parton model, as the nucleon momentum fraction carried by the struck parton.

resonance region under which predictions of the structure functions at large  $x$  lead to the same behavior as in the DIS region. (See Ref. [10] for a detailed review of quark-hadron duality).

Because of their different resonance spectra, it is expected, in certain theoretical models, that the onset of duality for the neutron will happen at lower momentum transfer than for the proton. Now that precise neutron spin-structure data [11] in the DIS region are available at large  $x$ , data in the resonance region are needed in order to test quark-hadron duality on the neutron spin-structure function  $g_1$ . The goal of experiment E01-012 was to provide such data on the neutron ( $^3\text{He}$ ) in the moderate  $Q^2$  region up to  $Q^2 = 4.0 (\text{GeV}/c)^2$  where duality is expected to hold.

In 2003, experiment E01-012 took data in Hall A at JLab. It was an inclusive measurement of longitudinally polarized electrons scattering off a longitudinally or transversely polarized  $^3\text{He}$  target [12]. Asymmetries and cross section differences were measured in order to extract the spin-structure function  $g_1$ :

$$g_1(E, E', \theta) = \frac{MQ^2\nu}{4\alpha_e^2} \frac{E}{E'} \frac{1}{E + E'} \left[ \Delta\sigma_{\parallel}(E, E', \theta) + \tan \frac{\theta}{2} \Delta\sigma_{\perp}(E, E', \theta) \right] \quad (1)$$

where the superscript  $\parallel$  ( $\perp$ ) represents the configuration between the incident electron longitudinal spin direction and the longitudinal (transverse) target spin direction. The quantities  $E$ ,  $E'$  and  $\theta$  correspond to the incident and scattered electron energies and the scattering angle, respectively. Also in Eq. 1,  $M$  is the mass of the target,  $\nu$  is the energy transfer to the target,  $\alpha_e$  is the fine structure constant. Note that our data allows a direct extraction of  $g_1$  (and  $g_2$ , see Eq. 7) without the need of an external input. All details on the experimental setup and the analysis steps can be found in [13].

The structure functions  $g_1$  and  $g_2$  were generated for the three incident energies and two scattering angles, and then, were interpolated to constant  $Q^2$ . In Fig. 1, the results from E01-012 on the spin-dependent structure function  $g_1^{^3\text{He}}(x, Q^2)$  (per nucleon<sup>2</sup>) are shown compared to parametrizations of parton distribution functions from four different groups [14, 15, 16, 17], taken at Next-to-Leading Order (NLO). Target-mass corrections were applied to the DIS parametrization following the prescription of Ref. [18]. These plots indicate that our resonance region data approach the DIS parametrizations with increasing  $Q^2$ .

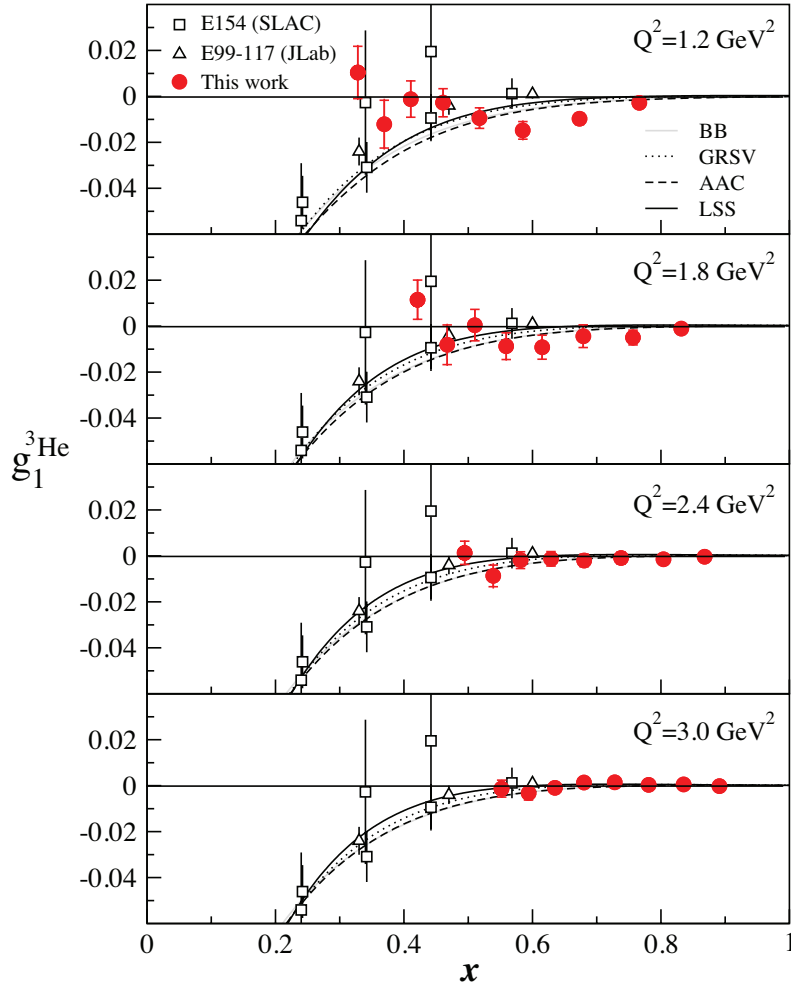
Note that the DIS parametrizations of  $g_1^{^3\text{He}}$  were generated using the proton and neutron  $g_1$  parametrizations and the effective polarization equation [20]:

$$g_1^{^3\text{He}} = P_n g_1^n + 2P_p g_1^p \quad (2)$$

where  $P_n = 0.86 \pm 0.02$  and  $P_p = -0.028 \pm 0.004$  are the effective polarizations of the neutron and the proton in  $^3\text{He}$ , respectively [21].

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<sup>2</sup> In Eq. 1, the proton mass was used instead of the mass of the  $^3\text{He}$  nucleus.

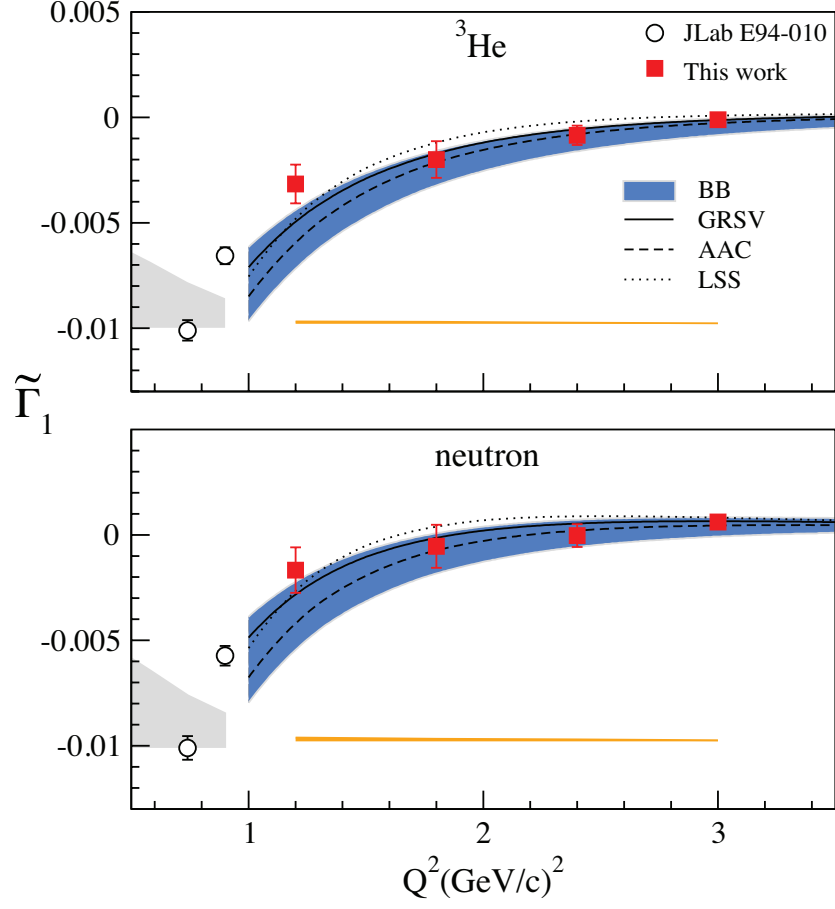


**FIGURE 1.** The spin-structure function  $g_1^{3\text{He}}$  in the resonance region at four  $Q^2$ -values. The error bars represent the total uncertainties with the inner part being statistical only. Also plotted are the DIS world data from experiments E154 at SLAC [19] and E99-117 at JLab [11] which are at different  $Q^2$  than our resonance data. The curves were generated from the NLO parton distribution functions of Ref. [14, 15, 16, 17] to which target-mass correction were applied

In order to quantitatively study quark-hadron duality in the spin-structure function  $g_1$ , a partial integration is performed:

$$\tilde{\Gamma}_1(Q^2) = \int_{x_{\min}}^{x_{\max}} dx g_1(x, Q^2) \quad (3)$$

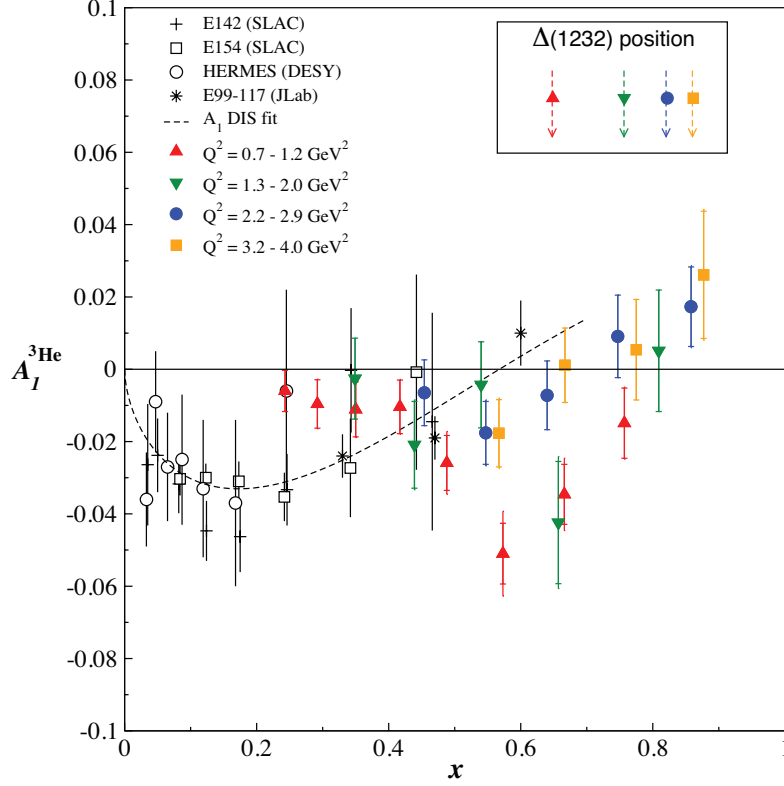
The partial moment for the neutron was extracted from the partial moment of  $^3\text{He}$  using Eq. 2 by replacing the  $g_1$ -quantities by their partial moments  $\tilde{\Gamma}_1$ . This procedure was shown to be valid in Ref. [22]. *Global* duality is defined as the partial moment over the entire resonance region, from pion threshold (with missing mass  $W = 1.079$  GeV corresponding to  $x_{\max}$ ) to  $W = 2.0$  GeV (corresponding to  $x_{\min}$ ). As for *local* duality, the partial integral is taken over a set of resonances.



**FIGURE 2.** The partial  $g_1$  first moment  $\tilde{\Gamma}_1^{^3\text{He}}$  and  $\tilde{\Gamma}_1^n$ : test of spin duality on  $^3\text{He}$  (top) and neutron (bottom). The recent data from E01-012 are plotted with red squares, with the error bars being statistical only and the orange band being the absolute systematic uncertainty. Also plotted are the DIS parameterizations of Blümlein and Böttcher [14] (blue band), GRSV [15] (solid curve), AAC [16] (dashed curve) and LSS [17] (dotted curve) after applying target-mass corrections. The open circles are data from JLab E94-010 [23] with the absolute systematic uncertainty represented by the grey band.

For all  $Q^2$  settings of E01-012, the data cover a  $x$ -range corresponding to a  $W$ -range extending from the pion threshold to  $W = 1.095$  GeV. Therefore we performed the integration of Eq. 3 over this  $x$ -range for our resonance data and for the DIS parameterizations shown in Fig. 1. The result of this quantitative test of quark-hadron duality is shown in Fig. 2. We can see a clear confirmation that global quark-hadron duality holds at least down to  $Q^2 = 1.8$  (GeV/c) $^2$  for  $^3\text{He}$  and the neutron. Note that global duality was experimentally observed for the proton and the deuteron spin structure functions [3, 4, 5] for  $Q^2$  above 1.7 (GeV/c) $^2$ .

We also studied quark-hadron duality on the virtual photon-nucleon asymmetry  $A_1$ , which can be expressed from the parallel and perpendicular asymmetries ( $A_{\parallel}$  and  $A_{\perp}$ ) as



**FIGURE 3.** The virtual photon-nucleon asymmetry  $A_1^{3\text{He}}$  in the resonance region. DIS data are from SLAC E142 [25], E154 [19], from DESY experiment HERMES [26] and from JLab E99-117 [11]. The error bars represent the total uncertainties with the inner part being statistical only. The curve represents a fit to the  $A_1^{3\text{He}}$  DIS data. The arrows in the black frame point to the  $\Delta(1232)$  peak position for each of our data sets

follows:

$$A_1 = \frac{A_{\parallel}}{D(1 + \eta\xi)} - \frac{\eta A_{\perp}}{d(1 + \eta\xi)} \quad (4)$$

The variables  $\eta$  and  $\xi$  depend on the kinematics, and  $D$  and  $d$  are functions of the longitudinal to transverse cross section ratio  $R(x, Q^2)$ . Details on our evaluation of  $R$  for  $^3\text{He}$  can be found in [24].

The virtual photon-nucleon asymmetry  $A_1^{3\text{He}}$  was extracted in the resonance region from our data at four different  $Q^2$ -ranges and is shown in Fig. 3. The position of the  $\Delta(1232)$  resonance is indicated for each subset of data. The most noticeable feature is the negative contribution of the  $\Delta(1232)$  resonance at low  $Q^2$ . It has been argued [7, 9] that quark-hadron duality should not work in the  $\Delta$ -region at low  $Q^2$ . However, at  $Q^2$  above 2.0 (GeV/c)<sup>2</sup>, the dominant negative bump at the location of  $\Delta(1232)$  seems to vanish. Furthermore the results from these higher  $Q^2$  settings show that the trend of  $A_1^{3\text{He}}$  goes to positive values with increasing  $x$ , as previously reported from the DIS world data. Our  $A_1^{3\text{He}}$  results from the two highest  $Q^2$  settings agree well with each other showing no strong  $Q^2$ -dependence.

The polarized  $^3\text{He}$  target was used in this experiment as an effective neutron target. Because of the dominant S-state of  $^3\text{He}$  where the two protons have their spins anti-aligned, we can expect neutron spin-structure functions to show similar behavior as observed for  $^3\text{He}$  structure functions here. Work is ongoing to extract the neutron  $A_1$  results from the  $^3\text{He}$  results using the new convolution approach of [27, 28].

## THE OTHER SPIN-STRUCTURE FUNCTION

In the naive parton model, the spin-structure function  $g_2$  does not exist. However the QCD parton model predicts a non-zero value for  $g_2$ . In the Operator Product Expansion (OPE) framework, both twist-two and higher twists operators contribute to  $g_2$  as follows:

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2) \quad (5)$$

where  $\bar{g}_2$  is the twist-three (and higher) contribution. The twist-two part of  $g_2$  can be expressed using the Wandzura-Wilczek formula defined entirely from the knowledge of the spin-structure function  $g_1$ :

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \quad (6)$$

The twist-three part of  $g_2$  is not  $1/Q$  suppressed compared to the  $g_2^{WW}$  (twist-two) part. Therefore  $g_2$  presents the unique advantage of the possible direct extraction of the twist-three contribution to the nucleon structure at high  $Q^2$ , where higher than twist-three contributions are suppressed.

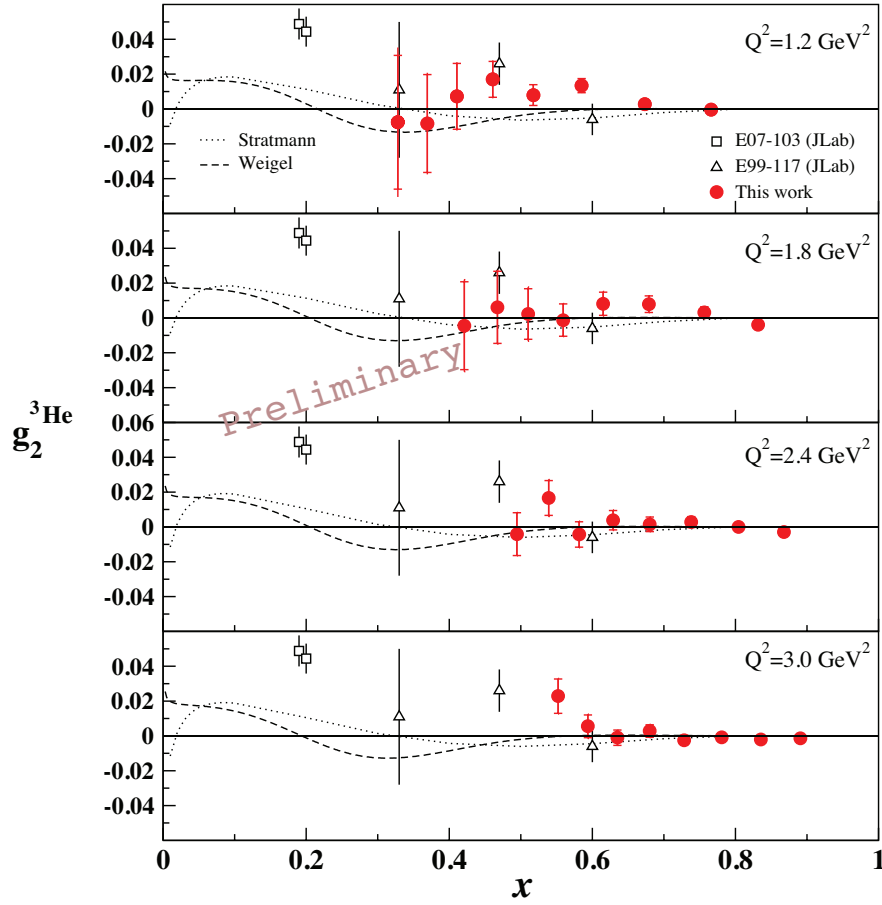
Experimentally, one can perform a model-independent measurement of  $g_2$  by scattering longitudinally polarized electron beam on a target with both longitudinal and transverse polarizations. The extraction of  $g_2$  from the polarized cross section differences is done following this formula:

$$g_2 = \frac{MQ^2v^2}{4\alpha_e^2} \frac{1}{2E'} \frac{1}{E+E'} \left[ -\Delta\sigma_{\parallel} + \frac{E+E'\cos\theta}{E'\sin\theta} \Delta\sigma_{\perp} \right] \quad (7)$$

Figure 4 presents the preliminary results on  $g_2^{^3\text{He}}$  from E01-012 at four  $Q^2$  values. Also plotted are calculations from chiral soliton model [30] and from the bag model [31] for  $g_2^{^3\text{He}}$  in the DIS region. In the  $x$ -range covered by our data, we can see that  $g_2^{^3\text{He}}$  is small and in agreement with the two theoretical models.

## The $x^2$ -weighted moment $d_2$

In the OPE framework [32, 33], information on the quark and gluon fields are contained in operators which can be *twist*-expanded in terms of  $1/Q^\tau$ . The twist  $\tau$  is defined as the mass dimension minus the spin of the operator. From here, several sum rules can be generated from the spin-structure functions  $g_1$  and  $g_2$ :



**FIGURE 4.** The spin-structure function  $g_2^{3\text{He}}$  (per nucleon) in the resonance region at four  $Q^2$ -values. The error bars represent the total uncertainties with the inner part being statistical only. Also plotted are the DIS world data from JLab experiments E97-103 [29] and E99-117 at JLab [11] which are at different  $Q^2$  than our resonance data. The dashed and dotted curves are calculations from the chiral soliton model [30] and from the bag model [31] respectively.

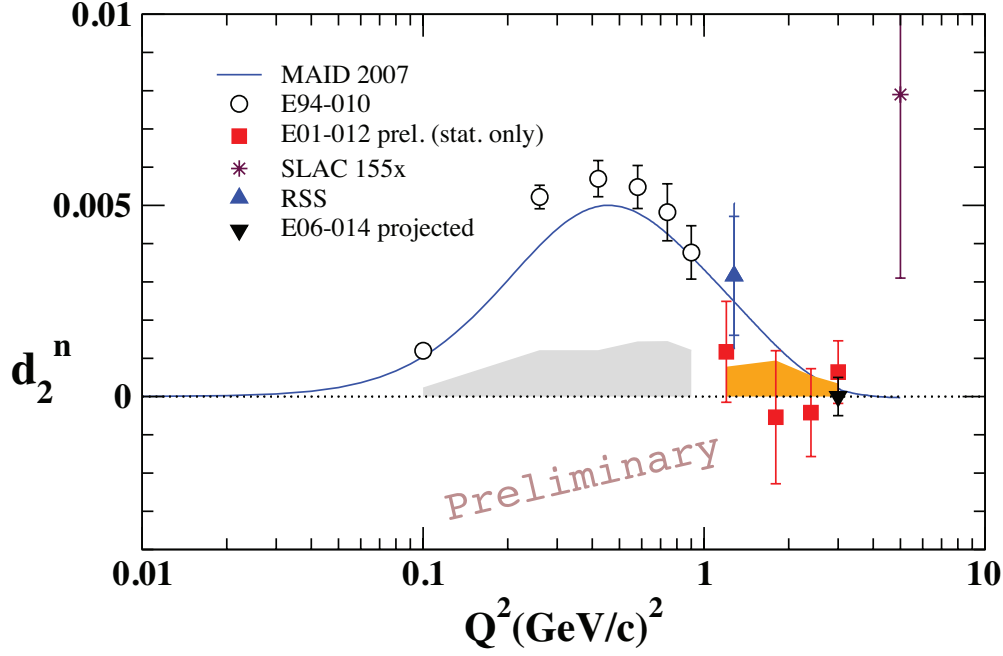
$$\int_0^1 dx x^n g_1(x, Q^2) = \frac{1}{2} a_n \quad n = 0, 2, 4, \dots \quad (8)$$

$$\int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{2} \frac{n}{n+1} (d_n - a_n) \quad n = 2, 4, \dots \quad (9)$$

with  $a_n$  ( $d_n$ ) are the twist-two (higher twists) reduced matrix elements. From Eqs. 8 and 9, we can extract the twist-three (and higher) matrix element  $d_2$ :

$$d_2(Q^2) = \int_0^1 dx x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] = 3 \int_0^1 dx x^2 \bar{g}_2(x, Q^2) \quad (10)$$

The leading twist quantities can be easily compared to naive parton model predictions. Higher twist effects are due to quark-quark and quark-gluon interactions. The twist-three quantities correspond to quark-gluon correlations.



**FIGURE 5.** Preliminary results on the *resonance contribution* to the neutron  $x^2$ -weighted moment  $d_2$  from E01-012. The error bars are statistical only and the band represents the experimental systematics. Data from JLab experiments E94-010 [23] and RSS [34] are shown. For comparison to the resonance contribution, we plotted the MAID model [35]. Also plotted are the *total*  $d_2$  from SLAC experiment E155x [36] and the projected result from JLab E06-014 [37], currently under analysis.

Fig. 5 shows the preliminary results for resonance region contribution to  $d_2^n$  from E01-012, and also from earlier JLab experiments E94-010 [23], RSS [34]. It is found to be very small for  $Q^2$  above 1 (GeV/c) $^2$ .

Prediction from lattice QCD calculation [38] has for the neutron  $d_2 = -0.001 \pm 0.003$  at  $Q^2 = 5$  and 10 (GeV/c) $^2$  with a  $Q^2$ -evolution close to constant down to  $Q^2 = 2$  (GeV/c) $^2$ . This could mean that the unmeasured part of  $d_2$  from E01-012 at  $Q^2 = 3$  (GeV/c) $^2$  would be also very small. JLab experiment E06-014 [37] should be able to tell us the answer in the next couple of years.

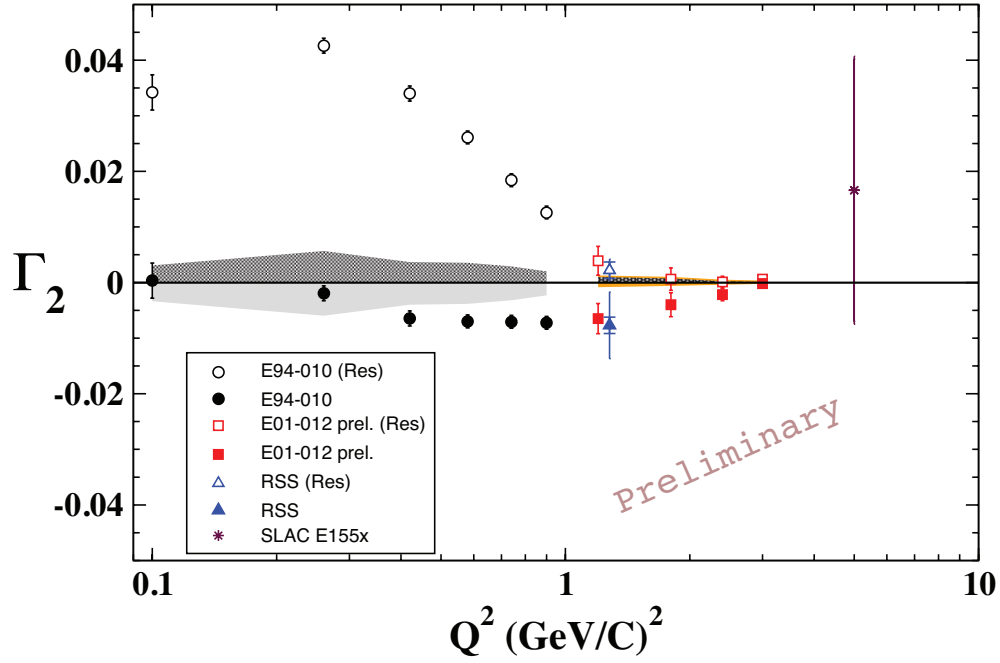
Also, it is really exciting to see the good agreement between E01-012 and RSS data since they come from two different experimental setups and two different targets: polarized  $^3\text{He}$  for E01-012 and polarized  $^2\text{H}$  for RSS.

### The Burkhardt-Cottingham Sum Rule

The Burkhardt-Cottingham (BC) sum rule [39] is a super-convergence relation derived from dispersion relation in which the virtual Compton helicity amplitude  $S_2$  falls off to zero more rapidly than  $\frac{1}{\nu}$  as  $\nu \rightarrow \infty$ . The sum rule is expressed as follows:

$$\Gamma_2(Q^2) \equiv \int_0^1 dx g_2(x, Q^2) = 0, \quad (11)$$





**FIGURE 6.** Preliminary results on the Burkhardt-Cottingham sum rule on the neutron from E01-012 (filled squares). The error bars are statistical only, the upper band represents the experimental systematics and the lower band the uncertainties on the unmeasured part of the sum rule. The open square data are the measured part of the integral as was performed by experiment E01-012. Also plotted are data from JLab experiments E94-010 [23] and RSS [34], with also the measured part of the integral represented by open symbols and the sum rule with filled symbols, and SLAC experiment E155x [36].

and is predicted to be valid at all  $Q^2$ . The validity of the sum rule derived through assumptions of Regge theory has been questioned [40]. Also it can be seen from Eq. 9 that the BC sum rule cannot be extracted from the OPE due to the non-existent  $n = 0$  expansion of  $g_2$ -moments.

Preliminary data from E01-012 on the BC sum rule are shown in Fig. 6. Also shown are data from JLab experiments E94-010 [23] and RSS [34]. All these experiments were concentrated on the resonance region and therefore have measured only the resonance part of Eq. 11. In order to generate the full integral, the unmeasured elastic and DIS contributions need to be added. For the elastic part, we used the parametrization from Ref. [41]. However, for the DIS contribution, we used  $g_2^{WW}$  which can be evaluated from our own  $g_1$  data. A conservative systematic uncertainty was associated with this approximation and more systematic studies are underway looking at using different theoretical models to evaluate the low  $x$  unmeasured part of the integral.

Nonetheless, at this point in our analysis, we can see in Fig. 6 data approaching the BC sum rule with increasing  $Q^2$ . We can also see the good agreement between E01-012 and RSS data.

## CONCLUSION

Experiment E01-012 provides spin-structure data in the resonance region for the neutron ( $^3\text{He}$ ) for  $1.0 < Q^2 < 4.0$  (GeV/c) $^2$  and  $0.30 < x < 0.85$ . Quark-hadron duality was found to hold globally for the neutron and  $^3\text{He}$  spin-structure function  $g_1$  at least down to  $Q^2 = 1.8$  (GeV/c) $^2$ . At  $x < 0.60$ , where DIS  $A_1^{^3\text{He}}$  data are available, a qualitative *local* test of quark-hadron duality was performed. The results show that  $A_1^{^3\text{He}}$  in the resonance region follows a similar behavior as  $A_1^{^3\text{He}}$  measured in the DIS region. The confirmation of quark-hadron duality for the neutron structure functions is important for a better understanding of the mechanism of quark-gluon and quark-quark interactions. Combined with already existing proton resonance data, a study of spin and flavor dependence of duality can be performed.

Preliminary results from E01-012 show small values for the neutron  $x^2$ -weighted moment  $d_2$  above  $Q^2 \approx 1$  (GeV/c) $^2$ . Also, our results on the Burkardt-Cottingham sum rule is in good agreement with the existing world data showing that the sum rule is valid at the two-sigma level for  $Q^2$  between 0.1 and 5.0 (GeV/c) $^2$ .

Finally, more results are expected to come from E01-012 as the extraction of  $A_1^n$  in the resonance from our data on  $^3\text{He}$ , the extended GDH sum rule, the Bjorken sum rule, etc.

## ACKNOWLEDGMENTS

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